Engineering Notes

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Improved Woodward's Panel Method for Calculating Edge Suction Forces

C. Edward Lan* and Sudhir C. Mehrotra†
University of Kansas, Lawrence, Kansas

Introduction

WOODWARD'S unified subsonic and supersonic panel method[1] with constant pressure panels has been in wide use for some time. The overall longitudinal aerodynamic characteristics predicted by the method are reasonably accurate. However, the predicted lifting-pressure distribution is not accurate enough for calculating the leading-edge and side-edge suction forces. The latter are quite sensitive to the paneling scheme and the control-point locations. In Ref. 2, a paneling scheme derived from numerical experimentation was suggested for calculating the leading-edge suction. Again, this approach is still not good enough for calculating the side-edge suction force. Examination of the two-dimensional flat-plate problem indicated that if a constant-percent control-point location is used, the overall force and moment and the pressure distribution cannot be accurately predicted simultaneously. Originally, Woodward[1] recommended 95% of the panel chord as the suitable location for the control point. Later, 85% location was suggested, based on matching the two-dimensional lift of a flat plate.[3,4] However, 85% location of control point does not produce accurate pressure distribution and pitching moment.

Recently, Dillenius and Nielsen[5] used the panel method to calculate the leading-edge and side-edge suctions in supersonic flow. After the strengths of the panel singularity were obtained, they were replaced by a system of equivalent horseshoe vortices. The in-plane forces are then calculated by applying the Kutta-Joukowski theorem and are assumed to be acting along the planform edges. Only a few supersonic results have been compared with other known theoretical calculations.

In this Note, a new technique to improve Woodward's panel method will be described. The pressure prediction is improved, based on a new two-dimensional theory. The method used in Ref. 6 is then applied to calculating the leading-edge and side-edge suction forces by directly using the predicted pressure distribution. The method is applicable in both subsonic and supersonic flow.

Theoretical Development

The main objective in the initial development is to predict exactly the pressure distribution of a flat-plate airfoil at locations at which the lift and moment coefficients can be easily obtained by integrating the pressure. According to the Quasi-Vortex-Lattice Method (QVLM),[7] these locations are best given by the cosine law distribution:

$$x_k = \{1-\cos[(2k-1)\pi/2N_c]\}/2, \; k=1,\ldots,N_c \tag{1}$$

where N_c is the number of chordwise panels. The chordwise paneling scheme will be based on the following cosine law:

$$x_{j+1} = [1-\cos(j\pi/N_c)]/2, \; j=0,1,\ldots,N_c \tag{2}$$

Assuming the angle of attack to be one radian, the airfoil integral equation can be written as

$$4\pi = \int_0^1 \frac{\Delta C_p(x')\,dx'}{x-x'} \tag{3}$$

If the chord is divided into N_c segments on each of which ΔC_p is assumed constant, Eq. (3) can be integrated over each segment to give

$$4\pi = \sum_{j=1}^{N_c} \Delta C_p(x_k) \ln|(x_j-x_i)/(x_{j+1}-x_i)|, \; i=1,\ldots,N_c \tag{4}$$

where x_i is the control-point location and $x_j<x_k<x_{j+1}$. Equation (4) will now be used to find x_i such that the predicted $\Delta C_{p_k}=\Delta C_p(x_k)$ will be exact. Since the problem is a nonlinear one, it can be solved by the following iterative method. Differentiating Eq. (4) with respect to x_i gives

$$\sum_{j=1}^{N_c} \frac{\partial \Delta C_{pk}}{\partial x_i} \ln\left|\frac{x_j-x_i}{x_{j+1}-x_i}\right| = -\sum_{j=1}^{N_c} \Delta C_{pk} \times\left[\frac{1}{x_{j+1}-x_i} - \frac{1}{x_j-x_i}\right], \; i=1,\ldots,N_c \tag{5}$$

Solving Eq. (5) will result in a set of $\partial\Delta C_{pk}/\partial x_i$- values. If $\Delta C_{pk}{}^{(d)}$ is the desired value and $\Delta C_{pk}{}^{(c)}$ is the computed one, an incremental change in control-point location Δx_i can be obtained from

$$\sum_{i=1}^{N_c} \frac{\partial \Delta C_{pk}}{\partial x_i}\Delta x_i = \Delta C_{pk}{}^{(d)} - \Delta C_{pk}{}^{(c)}, \; k=1,\ldots,N_c \tag{6}$$

If the starting x_i is chosen at 95% of panel chord, four or five iterations can produce an essentially exact solution. Once ΔC_{pk} are obtained, the lift coefficient is computed as

$$c_\ell = \int_0^1 \Delta C_p dx = \frac{1}{2}\int_0^\pi \Delta C_p \sin\theta d\theta \simeq \frac{1}{2}\frac{\pi}{N_c}\sum_{k=1}^{N_c}\Delta C_{pk}\sin\theta_k \tag{7}$$

A similar expression for c_m can be derived. With the predicted pressure distribution, the leading-edge suction can be easily calculated.

For three-dimensional wings, the chordwise distribution of control points and panels follows directly from the two-dimensional theory described above. The spanwise distribution of control stations and panel width is based on that used in the QVLM[7] in accordance with the cosine law.

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*Professor, Dept. of Aerospace Engineering. Associate Fellow AIAA.

†Research Associate, Dept. of Aerospace Engineering. Member AIAA.

The control stations are given by

$$y_i = (b/2)\{1-\cos[i\pi/(N_s+1)]\}/2, i=1,...,N_s \quad (8)$$

and the strip widths are given by

$$y_j = (b/2)\{1-\cos[(2j-1)\pi/2(N_s+1)]\}/2, j=1,...,N_s+1 \quad (9)$$

where N_s is the number of spanwise strips. The inboard gap created by the scheme given by Eq. (9) should be eliminated, and the tip inset from Eq. (9) is retained, as described in Ref. 7. In general, the control stations will not pass through the panel centroids. From extensive calculation, the chosen spanwise paneling scheme appears to be the best for calculating the side-edge suction force.

Once the pressure distribution is obtained, the interpolation by Fourier series is used to find the distribution of the leading-edge suction, the streamwise vortex density γ_x, and the side-edge suction. For example, to find the leading-edge suction parameter C, $(\Delta C_p \sin\theta)/2$ is developed in a cosine Fourier series:

$$f(\theta) = \frac{1}{2}\Delta C_p \sin\theta = a_0 + \sum_{j=1}^{N_c} a_j \cos j\theta \quad (10)$$

where the Fourier coefficients a_j can be computed by midpoint-trapezoidal rule in terms of ΔC_p. Near the leading edge, $\Delta C_p \cong C\sqrt{(1-x)/x}$. Hence,

$$\lim_{x\to 0} \frac{1}{2}\Delta C_p \sin\theta = C = a_0 + \sum_{j=1}^{N_c} a_j \quad (11)$$

The sectional leading-edge suction is then given by[2]

$$c_s = (\pi/8)C^2(1-M_\infty^2\cos^2\Lambda_{L.E.})^{1/2}/\cos^2\Lambda_{L.E.} \quad (12)$$

The side-edge suction per unit length of the tip chord is given by

$$s_t(x) = \pi\rho G^2(x) \quad (13)$$

where $G(x)$ is defined by

$$G(x) = \sqrt{\frac{b}{2}} \lim_{y\to b/2} \sqrt{1-y/(b/2)}\ \frac{1}{2}\gamma_x \quad (14)$$

and γ_x is the streamwise vortex density. To find γ_x, the conservation of vorticity is used:

$$\frac{\partial\gamma_x}{\partial x} + \frac{\partial\gamma_y}{\partial y} = 0 \quad (15)$$

If $\Gamma(x,y)$ is defined as

$$\Gamma(x,y) = -\int_{x_\ell}^{x} \gamma_y(x',y)\,dx' \quad (16)$$

Eq. (15) can be shown to give

$$\gamma_x = \frac{\partial\Gamma(x,y)}{\partial y} \quad (17)$$

In the linear theory, $\gamma_y = \Delta C_p/2$. Hence, Eq. (16) can be written as

$$\Gamma(x,y) = -\frac{c(y)}{2}\int_0^\theta \frac{1}{2}\Delta C_p \sin\theta\, d\theta \quad (18)$$

Using Eq. (10), Eq. (18) can be integrated in closed form. The differentiation in Eq. (17) is performed through the use of a trigonometric interpolation formula. The detail can be found in Ref. 6.

As can be seen from the above description, not only the leading-edge and side-edge suction forces can be predicted by the method, but also the distribution of γ_x which is needed in calculating some lateral-directional stability derivatives can be predicted.

Numerical Results and Discussions

For a flat-plate airfoil in incompressible flow, all aerodynamic characteristics of interest can be exactly predicted by the present improved method. The control points are in general located at 81.5% for the leading-edge panel and moved to 97.2% for the trailing-edge panel. For a cambered airfoil with or without flap deflection, the results are not exact, but quite accurate. The detail is given in Ref. 8.

One important application of the present method to three-dimensional planforms is the prediction of vortex lift through Polhamus' method of suction analogy.[9] According to this method, the total lift coefficient for a flat wing exhibiting edge-separated vortex flow can be written as

$$C_L = K_p \sin\alpha\cos^2\alpha + (K_{v,le} + K_{v,se})\sin^2\alpha\cos\alpha \quad (19)$$

where the lift factors K_p, $K_{v,le}$ and $K_{v,se}$ are the lift-curve slope, the leading-edge suction, and the side-edge suction coefficients at one radian of angle of attack, respectively, in accordance with the linear potential-flow theory. Thus, the accuracy of the present method in predicting the edge suction forces can be established by comparing the predicted lift factors with other theoretical results. Through extensive calculation, it was found that the present method has very good convergence characteristics with respect to the number of panels used. The results to be presented below were mostly obtained with nine chordwise elements and 12 spanwise strips. Since the chordwise control-point locations are based on the

Table 1 Comparison of predicted lift factors for various configurations

Geometry	M	K_p	$K_{v,le}$	$K_{v,se}$	Methods
$\Lambda = 0$ deg, $A = 2.0$	0	2.478	1.499	1.579	Present
$\lambda = 1.0$		2.476	1.500	1.549	Ref. 9
$\Lambda = 0$ deg, $A = 2.0$	1.5	2.737	0	1.109	Present
$\lambda = 1.0$		2.778	0	1.139	Exact[10,11]
		—	0	1.100	Ref. 5
$\Lambda = 0$ deg, $A = 3.5$	0.6	3.792	2.481	1.279	Present
$\lambda = 1.0$		3.778	2.480	1.250	Ref. 9
$\Lambda = 63$ deg, $A = 0.874$	0.6	1.325	1.641	1.525	Present
$\lambda = 0.4$		1.307	1.510	1.470	Ref. 9
$\Lambda = 45$ deg, $A = 1.0$	0	1.433	1.102	2.489	Present
$\lambda = 1.0$		1.431	1.101	2.412	Ref. 9

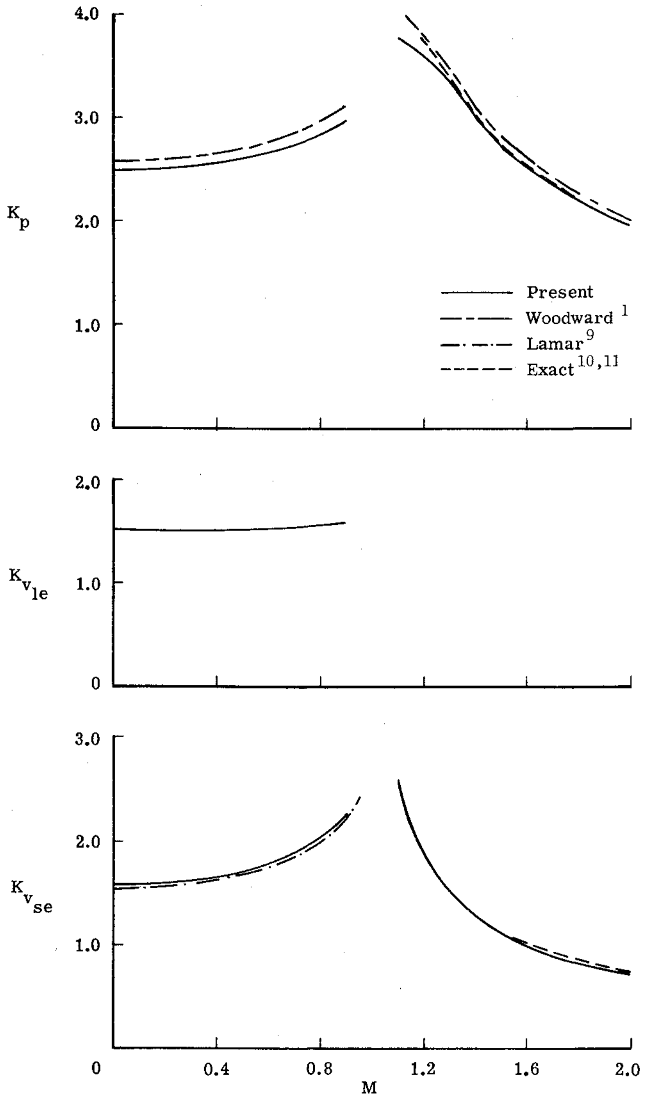

Fig. 1 Comparison of predicted lift factors for a rectangular wing of $A = 2.0$.

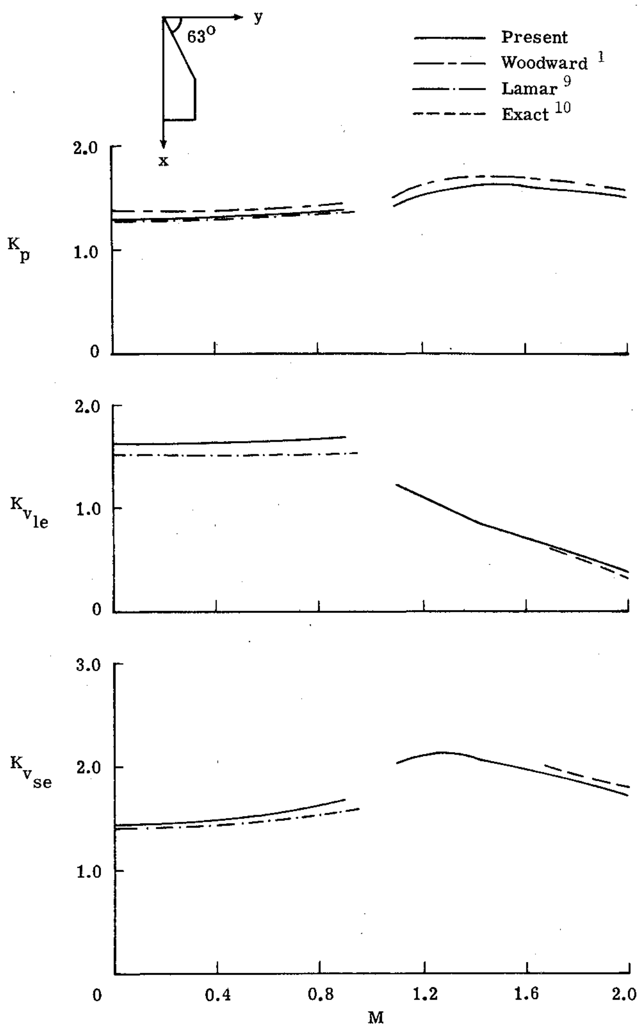

Fig. 2 Comparison of predicted lift factors for a cropped delta wing of $A = 0.874$, $\lambda = 0.4$.

two-dimensional incompressible flow theory, the predicted $K_{v,le}$ was found to be slightly too large in supersonic flow with subsonic leading edges and for highly swept planforms with leading-edge sweep angle, Λ, of 60 deg or higher. In these cases, the pressure distribution qualitatively will be different from the two-dimensional one. In supersonic flow, the problem was solved empirically by moving the control point of the leading-edge panel downstream by an additional 3% of the panel chord. If Λ is greater than 60 deg in subsonic and supersonic flow, the leading-edge panel control point is moved back by an additional 3.5% for every 10 deg over $\Lambda = 60$ deg. For variable-sweep wings, the sweep angle to be used is weighted with respect to the length of the leading edge.

Some typical results are presented in Table 1. As can be seen from this table, the present results agree quite well with Multhopp's method of Ref. 9 in subsonic flow, except for the cropped delta wing with $\Lambda = 63$ deg. For the latter case, the predicted $K_{v,le}$ is slightly higher, because the aforementioned correction of leading, control-point location for highly swept wings has not been applied. In Figs. 1 and 2, the variations of lift factors with Mach number for the rectangular wing of $A = 2.0$ and the cropped delta wing of $A = 0.874$ are presented. Both figures show that the original Woodward's method tends to predict higher K_p. In subsonic flow, the present results for K_p and $K_{v,le}$ for the rectangular wing are indistinguishable from those given by Ref. 9. In supersonic flow, the present results again agree reasonably well with available exact linear theory. Although only lift factors have been compared so far, the centroids of suction forces have also been found to be well predicted. More extensive comparison can be found in Ref. 8.

Concluding Remarks

The present improved Woodward's panel method has been shown to be capable of accurately predicting leading-edge and side-edge suction forces in subsonic and supersonic flow. Therefore, the method can be used not only to predict the vortex lift of complex planforms through the method of suction analogy, but also to calculate some lateral-directional stability derivatives as well.

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Effects of Dynamic Aeroelasticity on Aircraft Handling Qualities

Robert L. Swaim* and Wen-Yo Yen†
Purdue University, West Lafayette, Ind.

Introduction

THE subject of handling qualities requirements and criteria for highly elastic airplanes in turbulent and high dynamic pressure environments has been largely ignored by researchers. Much of the research on handling qualities has been concerned with relatively rigid, tactical military aircraft. The handling qualities parameters, such as phugoid, short-period, and dutch-roll frequencies and damping ratios, which have been determined pertinent for such airplanes, are considerably less meaningful for a flexible airplane with elastic mode frequencies close to the rigid body frequencies. When multiple frequencies are in close proximity, the pilot cannot easily discern individual modes of motion; rather, his opinion of the transient dynamics will likely be based on the time histories of the total motion.

No useful discussion of aeroelastic effects is included in the 1969 revision of the military aircraft handling qualities specification.[1] It contains only the following statement: "Since aeroelasticity, control equipment, and structural dynamics may exert an important influence on the airplane flying qualities, such effects should not be overlooked in calculations or analyses directed toward investigation of compliance with the requirements of this specification."

The specification is concerned mainly with desirable ranges of values on rigid body static and dynamic response parameters. There are methods available for estimating static aeroelastic corrections to rigid body aerodynamic stability derivatives; however, these are then used in rigid aircraft equations of motion. It seems quite possible that the desirable ranges of parameter values could be significantly affected by elastic mode degrees of freedom—particularly when some of the modes have natural frequencies of the same order of magnitude as the frequencies of the rigid body alone. It is not clear at all that the handling qualities should be specified by rigid body dynamic parameters when such mode interaction is present. In fact, the pilot would find it difficult to tell, for example, how much of a given pitch angle response to command input is due to rigid body and how much to low-frequency elastic modes.

Simplified Example

The total pitch angle time history that the pilot feels and sees, either on the outside horizon or the attitude indicator display, is given by:

$$\theta_i(x_p,t) = \theta(t) - \sum_{j=1}^{n} \phi_j'(x_p)\xi_j(t) = \theta(t) - \theta_e(t) \tag{1}$$

where x_p indicates pilot fuselage station, $\phi_j'(x_p)$ the slope of the jth symmetric elastic mode at that station, $\xi_j(t)$ the generalized displacement, $\theta(t)$ the rigid body pitch angle, and $\theta_e(t)$ the elastic contribution to total pitch angle (see Fig. 1). The relative contribution of the elastic terms to the total pitch angle increases as the natural frequencies of the elastic modes decrease. This can readily be seen by the following simplified example.

Equation (2) is the Laplace transformed longitudinal dynamics for a large flexible aircraft at a Mach 0.85, sea-level flight condition and includes the short-period, two-degree-of-freedom approximation plus the lowest frequency symmetric elastic mode. Equation (3) is the total pitch angle the pilot sees. The slope of the elastic mode is 0.025 at the cockpit.

$$\begin{bmatrix} s+1.21 & 0.94s & 0 \\ 0.48s+7.65 & s^2+1.57s & 0 \\ -2.26s+735.00 & 135.00s & s^2+1.14s+177.00 \end{bmatrix} \times \begin{bmatrix} \alpha(s) \\ \theta(s) \\ \xi_1(s) \end{bmatrix} = \begin{bmatrix} -0.29 \\ -15.18 \\ -2228.00 \end{bmatrix} \delta_e(s) \tag{2}$$

$$\theta_i(t) = \theta(t) - 0.025\xi_1(t) \tag{3}$$

The characteristic equation is:

$$s(s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2)(s^2 + 2\zeta_{1e}\omega_{1e}s + \omega_{1e}^2) = 0 \tag{4}$$

where the short-period, undamped natural frequency and damping ratio are $\omega_{sp} = 3.017$ rad/s and $\zeta_{sp} = 0.535$. The coupled elastic mode, undamped natural frequency, and damping ratio are $\omega_{1e} = 13.304$ rad/s and $\zeta_{1e} = 0.0428$. The in-vacuum, undamped natural frequency of the elastic mode is $\omega_1 = 13.59$ rad/s with structural damping of $\zeta_1 = 0.01$.

Figure 2 shows the rigid $\theta(t)$ and total $\theta_i(t)$ pitch angle responses to the up-elevator pulse shown in the figure. Notice

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*Professor, School of Aeronautics and Astronautics; presently, Associate Dean, College of Engineering, Oklahoma State University, Stillwater, Okla. Associate Fellow AIAA.

†Ph.D. Student; presently with Chung-Shan Institute of Science and Technology, Taiwan.